May 2015

YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Our next meeting

Saturday, June 6th 2015
3pm for 3.30pm

Seminar Room B.09
Parkinson Building
The University of Leeds

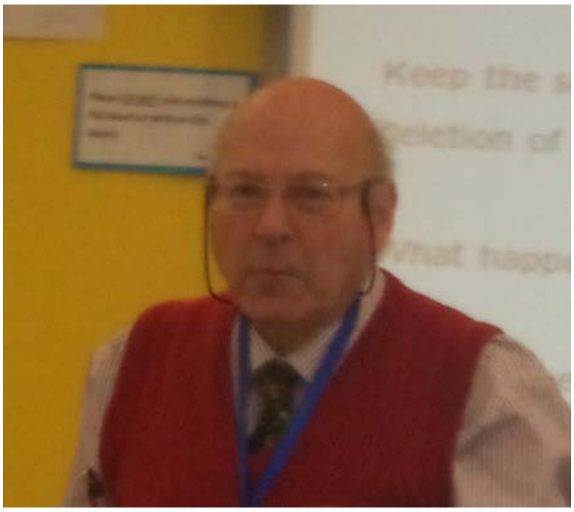
Dr Peter Neumann

How Hard Can Mathematics Be?

This talk will be based on a Masterclass designed to show pupils how much harder it is to factorize numbers than to multiply them. This leads into the beginnings of the subject called *computational complexity.*

Note the different venue

This meeting is a different venue from our recent meetings. It is in the basement of the Parkinson Building - the building with the prominent white tower at the University of Leeds. The room is accessible by a lift.



Peter Neumann at a UKMT Maths Circle in Wells, November 2013.

Peter Neumann is the President of the Mathematical Association. He was the founder Chairman of the United Kingdom Mathematics Trust and is distinguished both as a research mathematician, a teacher, and for his work on mathematical enrichment.

He is an Emeritus Fellow of The Queen's College, Oxford. He was awarded the OBE in 2008. His most recent book is *The mathematical writings of Évariste Galois.*

Peter Neumann is well known for his stimulating lectures on mathematics.

Non-members will be welcome at this meeting. Please bring it to the attention of your colleagues and friends and encourage them to come along. Our meetings are very friendly and include refreshments.

Anyone who would like to be added to our email list should send their name and email address to a.slomson@leeds.ac.uk

For more information about the Yorkshire Branch of the Mathematical Association, please go to our website http://ybma.org.uk.

Officers of the Yorkshire Branch of the Mathematical Association 2014-15

President: Alan Slomson (a.slomson@leeds.ac.uk)
Secretary: Bill Bardelang (rgb@bardelang.plus.com)
Treasurer: Tim Devereux (tim@devrx.org)

see overleaf for Mathematics in the Classroom.

A Rolling Circle

Consider a small circle inside a fixed larger circle, as shown in Figure 1. Let P be a point on the circumference of the smaller circle. What is the locus of P as the smaller circle rolls without slipping inside the larger circle?

The answer depends on the relative sizes of the two circles. Figure 2 shows the locus of P when the radius of the smaller circle is one-fifth that of the larger circle. What is the locus of P when the radius of the smaller circle is one-half that of the larger circle, as shown in Figure 3 ?

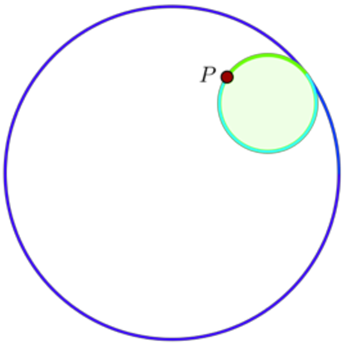


Figure 1

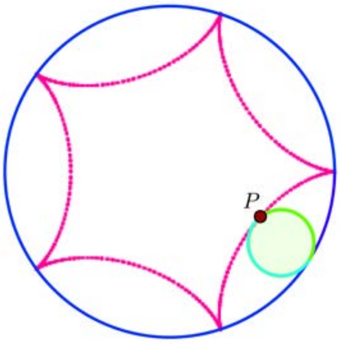


Figure 2

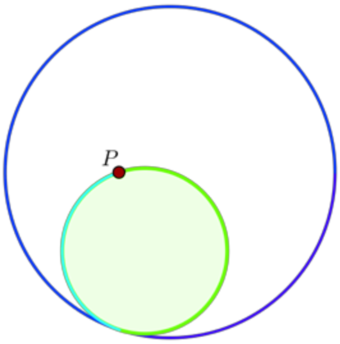
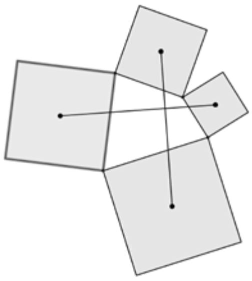


Figure 3

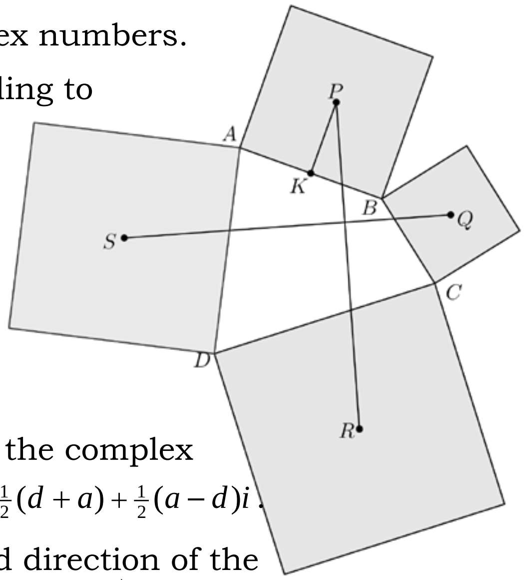
The January problem



In January we asked you what happens if you take any quadrilateral, draw a square on each side, and consider the line segments joining the centres of opposite pairs of these squares, as shown.

Van Aubel's Theorem says that these line segments are always equal in length and perpendicular. It is a special case of the Petr-Neumann-Douglas Theorem. The Neumann of this theorem, Bernard Neumann, was the father of our June speaker.

Van Aubel's Theorem can be proved using complex numbers.



Let a, b, c, d be the complex numbers corresponding to the vertices A, B, C, D of the quadrilateral. Then the midpoint, K, of AB corresponds to the complex number $\tfrac{1}{2}(a+b)$. KB has the magnitude and direction of $\tfrac{1}{2}(b-a)$. KP is obtained by rotating KB through $\tfrac{1}{2}\pi$ and so corresponds to $\tfrac{1}{2}(b-a)i$.

Therefore P corresponds to the complex number $\tfrac{1}{2}(a+b)+\tfrac{1}{2}(b-a)i$. Similarly Q, R, S correspond to the complex numbers $\tfrac{1}{2}(b+c)+\tfrac{1}{2}(c-b)i$, $\tfrac{1}{2}(c+d)+\tfrac{1}{2}(d-c)i$ and $\tfrac{1}{2}(d+a)+\tfrac{1}{2}(a-d)i$.

It follows that PR and QS have the magnitude and direction of the complex numbers $z=\left(\tfrac{1}{2}(c+d)+\tfrac{1}{2}(d-c)i\right)-\left(\tfrac{1}{2}(a+b)+\tfrac{1}{2}(b-a)i\right)$ and $w=\left(\tfrac{1}{2}(d+a)+\tfrac{1}{2}(a-d)\right)-\left(\tfrac{1}{2}(b+c)+\tfrac{1}{2}(c-b)i\right)$, respectively.

Since $z=\tfrac{1}{2}\left((-a-b+c+d)+(a-b-c+d)i\right)$ and $w=\tfrac{1}{2}\left((a-b-c+d)+(a+b-c-d)i\right)$, we have. $z=iw$ So PR and QS have the same lengths and are perpendicular.