April 2016

YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Our next meeting

Saturday, April 23rd 2016, 1:30pm for 2pm

MALL 1, School of Mathematics University of Leeds

Using iPads in the classroom

Paul Rowlandson
Trinity Academy, Halifax

In this session we will be exploring how iPads can be used to engage students, make learning enjoyably competitive and also support understanding. Those attending will put themselves in the shoes of a maths student by doing set activities and will also have time to explore freely the iPad apps towards the end.

This session will be followed at 4pm by the YBMA AGM.

Refreshments will be available.

Our June meeting

Saturday, June 11th 2016, 2pm for 2:30pm

MALL 1, School of Mathematics University of Leeds

Maths Mashup

Members of the YBMA have ten minute slots to show us some mathematics that they find particularly interesting.

We'd love to see you, and if you'd like to share, please feel free to bring some maths along with you.

We congratulate Alison Parker on the birth of her daughter, Natalie Melissa Pinto, who arrived safely at 6:01pm on Easter Sunday, 27th March. She weighed a healthy 7 lbs 9 oz (= 3.435kg), the same as her big sister.

Non-members will be welcome at these meetings. Please bring them to the attention of your colleagues and friends and encourage them to come along. Our meetings are very friendly and include refreshments.

Anyone who would like to be added to our email list should send their name and email address to a.slomson@leeds.ac.uk

For more information about the Yorkshire Branch of the Mathematical Association, please go to our website http://ybma.org.uk.

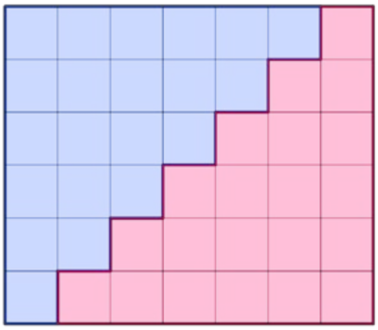
Officers of the Yorkshire Branch of the Mathematical Association 2015-16

President: Alan Slomson (a.slomson@leeds.ac.uk)
Secretary: Bill Bardelang (rgb@bardelang.plus.com)
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

see overleaf for Mathematics in the Classroom.

Mathematics in the Classroom

Proof without words



The diagram shows that a 6 × 7 grid of squares can be divided into two congruent polygons each made up of 1 + 2 + 3 + 4 + 5 + 6 squares. It follows that

$$1 + 2 + 3 + 4 + 5 + 6 = \frac{1}{2}(6 \times 7).$$

If we imagine that the diagram represents a grid which has size $n \times (n + 1)$, we see that it gives a proof without words of the formula

$$1 + 2 + \ldots + n = \frac{1}{2}n(n+1)$$

for the sum of the first n positive integers.

Another well known formula says that the sum of the first n cubes of the positive integers is the square of the sum of the first n positive integers. That is

$$1^3 + 2^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2.$$

Using the formula for the sum of the first n positive integers, we may rewrite this formula as

$$1^3 + 2^3 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2.$$

Can you find a proof without words of either version of this formula?

A curious integral

In the last Newsletter we posed the problem of what can be deduced from the value of the integral $I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$.

It can be checked that $\frac{x^4(1-x)^4}{1+x^2} = x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2}$. Therefore

$$I = \int_0^1 \left(x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2} \right) dx = \left[\frac{1}{7}x^7 - \frac{2}{3}x^6 + x^5 - \frac{4}{3}x^3 + 4x - 4\tan^{-1}x \right]_0^1$$

$$= \frac{1}{7} - \frac{2}{3} + 1 - \frac{4}{3} + 4 - 4\left(\frac{\pi}{4}\right) = \frac{22}{7} - \pi.$$ A surprise!

It is straightforward to check that $\int_0^1 x^4(1-x)^4\,dx = \frac{1}{630}$. For $0 \le x \le 1$, we have $1 \le 1 + x^2 \le 2$. Hence $\frac{1}{1260} = \int_0^1 \frac{x^4(1-x)^4}{2}\,dx \le \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx \le \int_0^1 x^4(1-x)^4\,dx = \frac{1}{630}$. Therefore $\frac{1}{1260} \le \frac{22}{7} - \pi \le \frac{1}{630}$ and so $\frac{22}{7} - \frac{1}{630} < \pi < \frac{22}{7} - \frac{1}{1260}$. This shows that $\frac{22}{7}$ gives quite a good approximation to π, using just some elementary calculus.

The discovery of this integral is credited to an engineer Donald Percy Dalzell (1898-1988) who published it in the paper D.P.Dalzell, *On* $\frac{22}{7}$, Journal of the London Mathematical Society, volume 19, 1944, pp133-134.