September 2016

YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Our next meeting

Tuesday, October 4th 2016, 7pm for 7:30pm

MALL 1, School of Mathematics

University of Leeds

The Retiring President's Talk

Counting in Circles

My favourite problem

by Alan Slomson

"The problem I will talk about is very rich in mathematical ideas, and makes an excellent resource for the classroom. I have used versions of it with school pupils from Year 6 upwards, mathematics undergraduates and at an MEI conference. It begins with some ideas which will be very familiar and takes in binomial coefficients, Euler's formula V + F = E + 2, and Cantor's transfinite numbers. It ends with what I believe to be a hard unsolved problem in number theory."

Coming soon …

Tuesday, November 8th 2016

A story of Mathematics told through postage stamps

by Jane Turnbull

A chance to follow the history of mathematics from prehistoric times to the modern day, illustrated with postage stamps from around the world. At this session, stamps, envelopes and postmarks will help depict the unfurling story.

Thursday, December 1st 2016

Christmas Quiz and Buffet

with Tim Devereux and Alan Slomson

Both meeting are in MALL 1, School of Mathematics, University of Leeds, at 7pm for 7.30pm.

A date for your new diary:
Wednesday, March 29th 2017, 2:30pm
W P Milne Lecture for Sixthformers
Dr Vicky Neale, University of Oxford
How to Solve Equations

Non-members are welcome at these meetings. Please bring them to the attention of your colleagues and friends and encourage them to come along. Our meetings are very friendly and include refreshments.

Anyone who would like to be added to our email list should send their name and email address to a.slomson@leeds.ac.uk

For more information about the Yorkshire Branch of the Mathematical Association, please go to our website http://ybma.org.uk.

Officers of the Yorkshire Branch of the Mathematical Association 2016-17

President: Lindsey Sharp (lindseyelizab50@hotmail.com)
Secretary: Alan Slomson (a.slomson@leeds.ac.uk)
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

see overleaf for Mathematics in the Classroom.

Mathematics in the Classroom

Divisibility Tests

The tests for divisibility by 2, 3, 4, 5, 8, 9 and 11 are well known. It does not seem to be so well known that there are also simple tests for divisibility by 7, 13, 17, 19, 23 and 29.

We give the test for divisibility by 29. We use the following notation. Given a number m, we let u be its units digit, and we let n be the number we obtain by deleting this digit from m. For example, if $m = 40252$, then $u = 2$ and $n = 4025$. In general, $m = 10n + u$, with $0 \le u \le 9$.

The divisibility test is as follows. We replace m by $n + 3u$. Then m is divisible by 29 if, and only if, $n + 3u$ is divisible by 29. This process is continued until we reach a two-digit number. For example, starting with 40252, we have:

$$40252 \to 4025 + 3 \times 2 = 4031 \to 403 + 3 \times 1 = 406 \to 40 + 3 \times 6 = 58$$

Because 58 is divisible by 29, we can deduce that 40252 is divisible by 29.

Here is another example

$$1663 \to 166 + 3 \times 3 = 175 \to 17 + 3 \times 5 = 32.$$

Because 32 is not divisible by 29, we can deduce that 1663 is not divisible by 29.

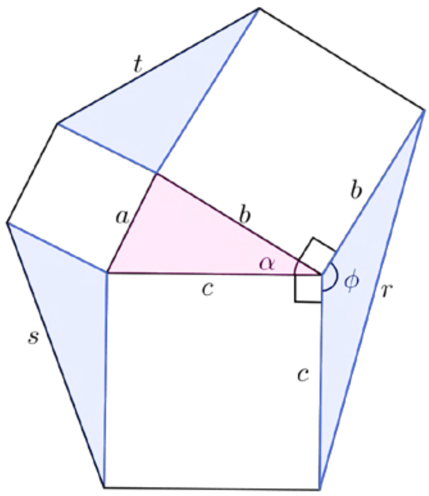
Explain why this test works. Can you find similar tests for divisibility by 7, 13, 17, 19 and 23?

Triangles and Squares

In the last Newsletter you were asked to prove that in the configuration shown in the diagram

$$r^2 + s^2 + t^2 = 3(a^2 + b^2 + c^2),$$

and we asked for the relationship between the areas of the blue triangles and the area of the pink triangle.



Solution

By applying the cosine formula to the pink triangle, we have,

$$a^2 = b^2 + c^2 - 2bc\cos\alpha . \qquad (1)$$

By applying the cosine formula to the triangle with sides of length b, c and r and with the angle ϕ, we have $r^2 = b^2 + c^2 - 2bc\cos\phi$. From the diagram, we see that $\alpha + \phi = 180^\circ$, and therefore, $\cos\phi = -\cos\alpha$. Hence, $r^2 = b^2 + c^2 + 2bc\cos\alpha$. [2]

Adding [1] and [2], we obtain $a^2 + r^2 = 2(b^2 + c^2)$, and therefore

$$r^2 = -a^2 + 2b^2 + 2c^2 \qquad [3]$$

Similarly $$s^2 = -b^2 + 2a^2 + 2c^2 \qquad [4]$$

and $$t^2 = -c^2 + 2a^2 + 2b^2 \qquad [5]$$

Adding [3], [4] and [5] we deduce that $r^2 + s^2 + t^2 = 3(a^2 + b^2 + c^2)$,

The area of the blue triangle with the angle ϕ is $\frac{1}{2}bc\sin\phi$. Because $\alpha + \phi = 180^\circ$, $\sin\alpha = \sin\phi$. Therefore $\frac{1}{2}bc\sin\phi = \frac{1}{2}bc\sin\alpha$. So the blue triangle has the same area as the pink triangle. Similarly, the other blue triangles have the same area as the pink triangle.