March 2017

YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Our next meeting

Saturday, 11 March 2017,

1.30pm for 2pm

The Mall, School of Mathematics
University of Leeds

Study Session

Problem Solving - so what is new?

Colin Prestwich
Yorkshire Ridings Maths Hub

How to develop problem solving strategies in the classroom by developing the teachers and supporting them with PD and resources.

Does lesson study have a distinct role?

How can the new Core maths level 3 award support this?

What about assessment and links to handling data and experimentation.

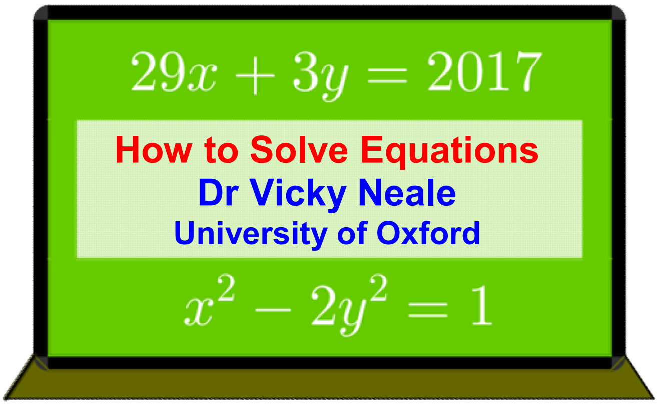
Thoughts from a serial head of maths and maths hub lead.......

This meeting will be followed by the YBMA AGM (at the same venue): a short formal meeting.

Refreshments will be provided.

W P Milne Lecture for Sixthformers

Wednesday, 29 March 2017, 2.30pm



Vicky Neale is Whitehead Lecturer in the Mathematical Institute, Oxford, and a Fellow of Balliol College. She has appeared frequently in Melvyn Bragg's *In Our Time* and other programmes.

She is well known for her excellent talks explaining mathematics to young people.

To book places for schools go to www.leeds.ac.uk/festival of science. and use the online booking form. Enquiries to R.M.Holland@leeds.ac.uk.

This talk is also open to YBMA members, who do not need to book in advance.

Officers of the Yorkshire Branch of the Mathematical Association 2016-17

President: Lindsey Sharp (lindseyelizab50@hotmail.com)
Secretary: Alan Slomson (a.slomson@leeds.ac.uk)
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)
see overleaf for Mathematics in the Classroom.

Non-members are welcome at these meetings. Please bring them to the attention of your colleagues and friends and encourage them to come along. Our meetings are very friendly and include refreshments.

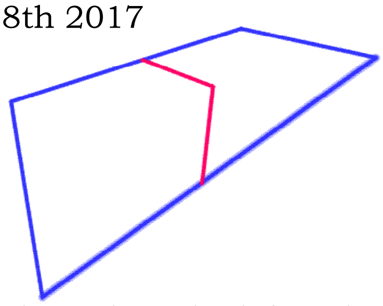
Anyone who would like to be added to our email list should send their name and email address to a.slomson@leeds.ac.uk

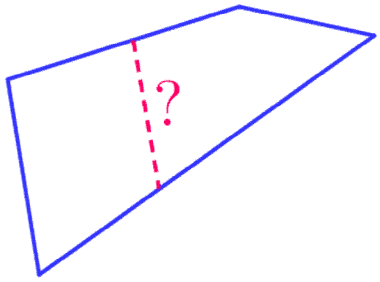
For more information about the Yorkshire Branch of the Mathematical Association, please go to our website http://ybma.org.uk.

Mathematics in the Classroom

Dividing a Quadrilateral

This problem was posed by Dr Jennie Golding in her talk to the Branch on February 8th 2017



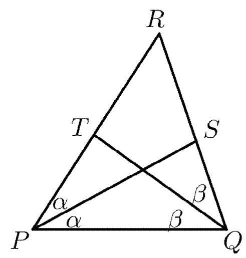


The quadrilateral on the left is divided into two pentagons by the broken line shown in red.

Can you construct a single straight line which divides the quadrilateral into two quadrilaterals with areas in the same ratio as the pentagons in the left hand diagram?

Lehmus's Angle Bisector Problem

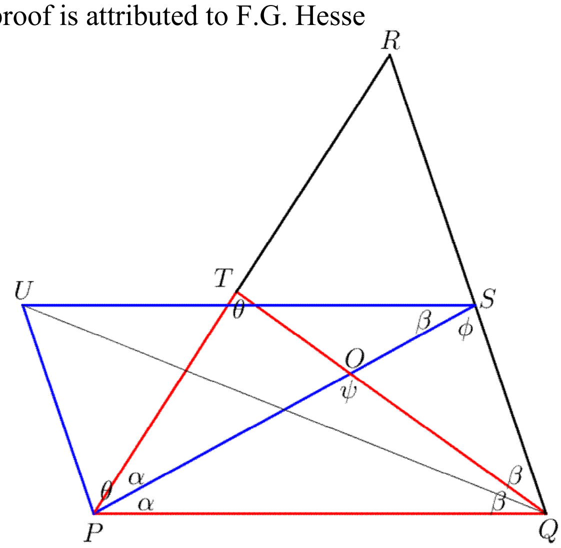
In the last Newsletter we asked for a proof of the following theorem.



Suppose that in the triangle *PQR*, the line bisecting the angle *QPR* meets *RQ* at *S*, and the line bisecting the angle *RQP* meets *RP* at *T*, with *PS* having the same length as *QT*.

Then the triangle *RPQ* is isosceles with *RP* = *RQ*.

We take the proof below from the article *The Steiner-Lehmus Angle Bisector Theorem* by John Conway and Alex Ryba, in the book *The Best Writing on Mathematics 2015* edited by Mircea Pitici, published Princeton University Press in 2016. There the proof is attributed to F.G. Hesse



Because $PS = QT$, it is possible to construct a triangle *PSU* that is congruent to the triangle *QTP*. So that $\angle PSU = \angle PQT = \beta$ and $\angle UPS = \angle QTP = \theta$, say.

We let *O* be the point where *PS* meets *QT*, and $\angle QOP = \psi$. From the triangle *PQR* we can deduce that $2\alpha + 2\beta$ is less than 180°. Therefore $\alpha + \beta$ is less than 90°. It follows that ψ is greater than 90°.

Because ψ is an external angle of the triangles *OPT* and *OQS* we have $\psi = \alpha + \theta$ and $\psi = \beta + \phi$.

We now compare the triangles *QUP* and *QUS*.

These triangles have the side *QU* in common. Because the triangles *PSU* and *QTP* are congruent, $PQ = US$. Also, $\angle UPQ = \alpha + \theta = \psi = \beta + \phi = \angle QSU$.

It follows that the triangles *QUP* and *QUS* are congruent. Therefore $PU = QS$. It follows that *PQSU* is a parallelogram. Because *SU* is parallel, $\alpha = \beta$. Hence $\angle RPQ = 2\alpha = 2\beta = \angle RQP$. It follows that the triangle *RPQ* is isosceles.

Note: You may think we have use the illegitimate *ASS* condition to show that the triangles *QUP* and *QUS* are congruent. However, we have shown that the common non-included angle ψ is obtuse. The *OSS* condition – two equal sides and equal non-included obtuse angles – is a valid congruence condition. Think about it!