October 2017

YBMA News

Vol. 27 No. 1

The Newsletter of the Yorkshire Branch of the Mathematical Association

Our next meeting

Wednesday, 11 October 2017,

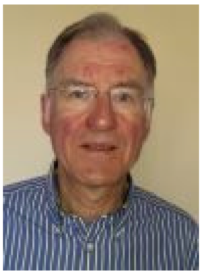
7pm for 7.30pm

The Mall, School of Mathematics
University of Leeds

Mary, Mary, quite contrary, how does your garden grow?
With silver bells and cockle shells and Fibonacci all in a row

Tom Roper

President of the Mathematical Association



"2016 saw the RHS Chelsea Flower Show feature a Mathematics Garden and whilst I didn't go and see it, the articles about it that I read caused me to explore a little. The talk and activities therein won't improve your garden but it might add something to your mathematical diet."

Refreshments.

Dates for your diary

Thursday, December 7th 2017
7pm for 7.30pm
Christmas Quiz and Buffet
A sociable evening of brain-frazzling questions, quirky mathematical prizes with seasonal food and drink. Cajole your colleagues to come and enjoy themselves.

Saturday, March 10th 2018
11am-3pm
Enriching the teaching of A-level mathematics: a study day for teachers.

Tom Roper
Teaching Mechanics: assumptions, misconceptions and Newton's Laws

Lizzie Kimber
Using rich tasks for teaching A-level Mathematics

Wednesday, March 21st 2018
2.30pm-3.30pm
W.P.Milne Lecture for Sixth Formers
Are large databases good for your health?
Dr Paul Baxter, Associate Professor in Biostatistics, University of Leeds

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Officers of the Yorkshire Branch of the Mathematical Association 2016-17
President: Lindsey Sharp (lindseyelizab50@hotmail.com)
Secretary: Alan Slomson (a.slomson@leeds.ac.uk)
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)
see overleaf for Mathematics in the Classroom.

Non-members are welcome at these meetings. Please bring them to the attention of your colleagues and friends and encourage them to come along. Our meetings are very friendly and include refreshments.

Anyone who would like to be added to our email list should send their name and email address to a.slomson@leeds.ac.uk

For more information about the Yorkshire Branch of the Mathematical Association, please go to our website http://ybma.org.uk.

Mathematics in the Classroom

Approximations to π using continued fractions.

It is well known, but not easy to prove, that the number π that gives the ratio of the circumference of a circle to its diameter is an irrational number. Indeed, although this is often glossed over, it is not immediately obvious that this ratio is the same for all circles whatever their size.

Also well known is that π is approximated quite well by the rational number $\frac{22}{7}$. It is not quite so well known that the rational number $\frac{355}{113}$ gives a surprisingly good approximation to π. With a denominator of just over 100, we expect an approximation that is only accurate to two decimal places. However, we have π = 3.141 592 653 … while $\frac{355}{113}$ = 3.141 592 920 …, and hence gives an approximation which is accurate to 6 decimal places.

How can we find good rational approximations to π from its decimal expansion? Continued fractions provide the answer. We can illustrate the process as follows:

$$3.141\,592\,653\ldots = 3 + 0.141\,592\,653$$

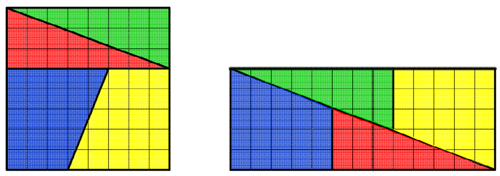
$$= 3 + \frac{1}{1/0.141\,592\,653\ldots} = 3 + \frac{1}{7.062\,513\ldots}$$

$$= 3 + \cfrac{1}{7 + \cfrac{1}{1/0.062\,513\ldots}} = 3 + \cfrac{1}{7 + \cfrac{1}{15.996\,594\ldots}}$$

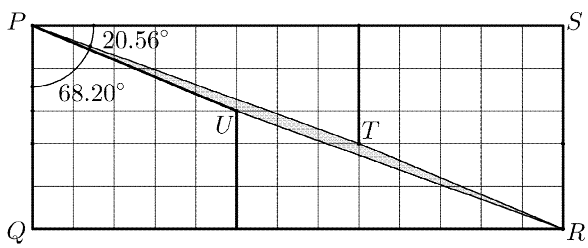
$$= 3 + \cfrac{1}{7 + \cfrac{1}{15 + 0.996\,594\ldots}}.$$

The process consists of repeatedly subtracting the integer part of the current number, and taking the reciprocal of the fractional part. Because π is irrational, the process continues indefinitely. If we truncate the continued fraction we obtain rational approximations to π which get more and more accurate as we take more and more terms. For example, we find that $3 + \cfrac{1}{7 + \cfrac{1}{15}} = 3 + \cfrac{1}{\frac{106}{15}} = 3 + \frac{15}{106} = \frac{333}{106}$. It is left as an exercise to show that the next truncation gives the approximation $\frac{355}{113}$. Pupils should then be encouraged to use the same method to find good rational number approximations to other irrational numbers such as $\sqrt{2}$ and $\sqrt{3}$.

A Geometrical Paradox



It looks from the figure that an 8 × 8 rectangle has been dissected and the parts rearranged to make a 13 × 5 rectangle. In the last issue we asked where the extra 1 unit of area comes from.



The answer is that the above figure is misleading. The diagonal of the 13 × 5 rectangle does not coincide with the sides of the trapeziums and triangles. This is shown in the figure on the left in which the discrepancy has been exaggerated. It can be checked that the parallelogram *PURT* has area 1.

Notice that the numbers 5, 8, 13 that occur here are consecutive Fibonacci numbers. The relevant fact is that $5 \times 13 = 8^2 - 1$. How does this generalize?