May 2018

YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Our next meeting Saturday 9th June 2018 at 2.30pm

The Mall
School of Mathematics
University of Leeds

A story of Mathematics told through my stamp collection

Jane Turnbull

"A chance to have a look at some of my stamp (and other related philatelic items) collection which tells a history of mathematics. You will be able to hear about a selection of my favourite bits and for those of you who enjoyed the talk last year there will definitely be a new selection for you to see."

Followed by the YBMA AGM at the same venue.

Refreshments will be provided

Officers of the Yorkshire Branch of the Mathematical Association 2017-18

President: Lindsey Sharp (lindseyelizab50@hotmail.com)

Secretary: Alan Slomson (a.slomson@leeds.ac.uk)

Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

See overleaf for *Mathematics in the Classroom*

Mathematics in the Classroom

What difference does a diagram make?

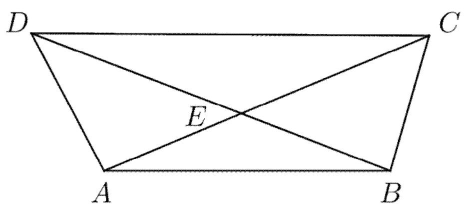
The Problem from the previous Newsletter

Let *ABCD* be a convex quadrilateral. Let *E* be the point where the diagonals *AC* and *BD* meet. The triangles *AEB* and *DEC* have the same area.

Prove that *AD* is parallel to *BC*.

We deliberately did not give a diagram for this problem, as we conjectured is that the way the problem is solved will depend on which diagram is drawn.

Method 1



Since the triangles *AEB* and *DEC* have the same area

$$\tfrac{1}{2}(AE \times EB \times \sin \angle AEB) = \tfrac{1}{2}(DE \times EC \times \sin \angle DEC).$$

Therefore, because the vertically opposite angles $\angle AEB$ and $\angle DEC$ are equal,

$$AE \times EB = DE \times EC .$$

It follows that

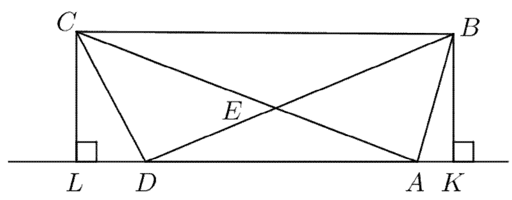
$$\frac{AE}{DE} = \frac{EC}{EB} .$$

We can deduce that the triangle AED is similar to the triangle CEB . Hence

$$\angle ADE = \angle CBE .$$

It follows that *AD* is parallel to *BC*.

Method 2



Since the triangles *AEB* and *DEC* have the same area, by adding to each triangle the triangle *AED* we deduce that the triangles *ABD* and *ACD* have the same area.

These triangles share *AD* as their base, they have the same height. That is, $BK = CL$, where *K*, *L* are the feet of the perpendiculars from *B* and *C*, respectively, to the line through *A* and *D*.

It follows that *KBCL* is a rectangle.

Therefore *KL* is parallel to *BC*. We deduce that *AD* is parallel to *BC*.

This second method seems very natural when the diagram is drawn so that DA is horizontal, but less so with the first diagram.

A Regular Generalization

ABCDE is a regular pentagon.

Find the angles *EAD*, *DAC* and *CAB*.

What do you notice?

How may this be generalized?