December 2018

# YBMA News

Vol. 28 No. 2

The Newsletter of the Yorkshire Branch of the Mathematical Association

## Our next meeting

**Tuesday, December 4th at 7.30pm**

**School of Mathematics University of Leeds**

## Our famous Christmas Quiz

**devised by Tim Devereux and Alan Slomson**

**with seasonal refreshments and prizes for all!**

**Bring your friends and colleagues!**

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Missing minutes

Thanks to Tony Orton we have the YBMA minute books from the foundation of the Branch in 1920 to September 1984. If you have copies of any later minutes, please contact Alan Slomson, a.slomson@leeds.ac.uk

**Officers of the Yorkshire Branch of the Mathematical Association 2018-19**

*President:* Lindsey Sharp (lindseyelizab50@hotmail.com)

*Secretary:* Alan Slomson (a.slomson@leeds.ac.uk)

*Treasurer:* Jane Turnbull (da.turnbull@ntlworld.com)

See overleaf for *Mathematics in the Classroom*

## Dates for your diary

**Saturday, 9th March 2019**
**Enriching the teaching of A-level Statistics: a Study Day for Teachers**
led by **Stella Dudzic** and **Darren Macey**

**Wednesday, 3rd April 2019 at 2:30pm**
**W.P.Milne Lecture for sixthformers**
***To Infinity and Beyond***
**Dr Katie Chicot**

**Full details of these events are attached.**

**Please bring them to the attention of your colleagues and anyone who might be interested in them.**

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### MA Branches Committee Bursary worth £499 to attend Annual Conference

One bursary of £499 (early price) is offered to cover the full residential cost of attendance at the 2019 Joint MA/ATM Conference at Chesford Grange, Warwick. The dates of the conference are 15th to 18th April 2019.

Applications are encouraged from anyone who is (a) a first time delegate at a MA conference, (b) a paid up member of the MA before 31 March 2019 and also (c) prepared to write a report on their experiences at the conference for publication in MA News, MA website, etc.

Application is by email direct to the Branches Committee Chair, Keith Cadman, at keithcadman@blueyonder.co.uk (copied to Marcia Murray at senioradministrator@m-a.org.uk).

Applicants should give their name, contact details, role in education and a response to the question "*How would attendance at this conference benefit you in performing your role?*".

Applications must be received no later than the end of December 2018. The successful applicant will be notified by 15 January 2019.

# Mathematics in the Classroom

## Integers written in terms of squares.

We note that

$$1 = 1^2,$$
$$2 = -1^2 - 2^2 - 3^2 + 4^2,$$
and $$3 = -1^2 + 2^2.$$

Now write 4, 5, 6, …. in a similar way as a combination of the first $k$ squares, for some positive integer $k$.

In fact, every integer, positive, negative and 0, may be written in this form. That is, each integer may be written as

$$\pm 1^2 \pm 2 \pm 3^2 \pm \ldots \pm k^2$$

for a suitable choice of $k$ and plus and minus signs.

Can you prove this?

### How many squares are triangular?

In the last Newsletter we noted that the number 1 is both a square and a triangular number. We asked whether there are any more squares that are also triangular numbers. We also asked the more difficult question: "How many squares are there which are also triangular numbers?"

The first three triangular squares after 1 are

$36 = 6^2 = \frac{1}{2}(8 \times 9)$ , $1225 = 35^2 = \frac{1}{2}(49 \times 50)$, and $41616 = 204^2 = \frac{1}{2}(288 \times 289)$.

[Recall that the formula for the $n$th triangular number is $\frac{1}{2}(n \times n+1)$.]

We now indicate a proof that there are infinitely many triangular squares.

The triangular squares are given by the positive integer solutions of the equation

$$m^2 = \tfrac{1}{2}(n \times n+1).$$

The integers $n$ and $n+1$ are coprime. Hence, for $\frac{1}{2}(n \times n+1)$ to be a square, either $n$ and $\frac{1}{2}(n+1)$ are both squares, or $\frac{1}{2}n$ and $n+1$ are both squares.

In the first case, for some positive integers $x$ and $y$, $n = x^2$ and $\frac{1}{2}(n+1) = y^2$. This gives $x^2 + 1 = 2y^2$, that is,

$$x^2 - 2y^2 = -1 \qquad (1)$$

In the second case, for some positive integers $x$ and $y$, $n+1 = x^2$ and $\frac{1}{2}n = y^2$ which gives $x^2 - 1 = 2y^2$, that is,

$$x^2 - 2y^2 = 1 \qquad (2)$$

Equations (1) and (2) are cases of Pell's equation. It is known that the terms of the infinite sequence defined by $(x_1,y_1) = (1, 1)$ and for $n > 1$,

$(x_{n+1},y_{n+1}) = (x_n + 2y_n, x_n + y_n)$ give, alternately, all the solutions of (1) and (2).

Each of these infinitely many solutions corresponds to a square which is also a triangular number. For example, you can check that $(x_2,y_2) = (3, 2)$, giving $n = x_2{}^2 - 1 = 8$ and hence $m^2 = \frac{1}{2}(n \times n+1) = \frac{1}{2}(8 \times 9) = 36$.